Indian Statistical Institute, Bangalore

B. Math. (hons.) Second/Third Year, Second Semester

Ordinary Differential Equations

Mid Term Examination — Date : 22 February 2023

Maximum marks: 30 — Time: 2hours

Answer any six, each question carries 5 marks.

1. Prove that $Mdx + Ndy = 0$ is exact if and only if $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ and use it to solve $(\sin x \sin y - xe^y)dy = (e^y + \cos x \cos y)dx$.
2. Let y_1 and y_2 be two linearly independent solutions of $y'' + P(x)y' + Q(x)y = 0$ on $[a, b]$. Show that $P = \frac{y_2 y_1'' - y_1 y_2''}{W(y_1,y_2)}$ and $Q = \frac{y_1' y_2'' - y_1'' y_2'}{W(y_1,y_2)}$.
3. Let p and q be constants. Reduce $x^2y'' + xpy' + qy = 0$ to a linear equation with constant coefficients and use it to solve $x^2y'' + 2xy' - 12y = 0$.
4. Solve the system $x' = 3x - 4y$ and $y' = x - y$.
5. If y is a nonzero solution of $y'' + Py' + Qy = 0$ on $[a, b]$ where P and Q are continuous functions on $[a, b]$. Prove that $\{x \in [a, b] \mid y(x) = 0\}$ is a finite set.
6. Solve $y'' + xy = 0$ in terms of power series of x.
7. Does $2xy'' + (3 - x)y' - y = 0$ has two independent Frobenius series solutions? Justify your answer.
8. (a) Prove $\frac{d(x^p J_p(x))}{dx} = x^p J_{p-1}(x)$ (*marks 3*).

   (b) Prove that between two positive zeros of J_p, J_{p-1} has a zero.